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RDT & E Project No. 1M010501A006

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

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ABSTRACT

In earlier attempts to describe the penetration of shaped-charge jets, the total depth of penetration was computed from $\int U dt$. Evaluation of the integral has not been possible because the penetration velocity U for the discontinuous portion of the jet has not been properly defined. In the following analysis, it is postulated that the total penetration can be represented by $\int U dt + \sum \Delta P$, where the integral is taken over the penetration produced by the continuous portion of the jet and the summation is taken over the penetration produced by the discrete particles resulting from jet breakup. It is shown that numerical values of the penetration variables for the 105mm unconfined test charge are compatible with the modified penetration equation. Scaling relations are introduced, and the penetration equations are given in size independent variables, which are substantiated experimentally. Although it was assumed that the incompressible approximation provides an adequate relationship between the jet and penetration velocities for a continuous jet, it is later shown that the finite compressibilities of the jet and target materials do not seriously alter this relation. The incompressible approximation does underestimate the pressure by an appreciable amount.

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INTRODUCTION

A shaped charge is a jet-forming explosive device containing an axially symmetric cavity lined with a thin metal liner. The geometrical design of a shaped charge can be varied over a wide range; however, cylindrical charges containing conical cavities have been most extensively investigated. Line drawings illustrating the jet formation and penetration process for a typical shaped charge are presented in Figure 1. When the explosive is detonated, the high pressure of the detonation products collapses the metal liner upon itself forming a high-velocity, forward-moving, metallic jet, which is able to penetrate high-strength materials such as armor.[1] Different elements of the liner experience different collapse velocities and the resulting jet contains a spectrum of velocities with forward portions of the jet traveling faster than succeeding portions.[2,3] The velocity gradient in the jet causes it to lengthen as it travels forward from the charge. Initially the jet lengthens in a ductile manner, but eventually the ductility of the material is exceeded and the jet fractures into a series of discrete particles.

The pressure exerted by the impinging jet greatly exceeds the yield strength of the target material, and penetration takes place by a hydrodynamic process as illustrated in Figure 1. Theories describing the penetration of a shaped-charge jet were advanced independently by Pugh[1] in the United States and by Hill, Mott, and Pack* in Great Britain. Both theories were based on steady-state incompressible fluid flow and resulted in an equation of the form

$$\lambda \rho_j (V_j - U)^2 = \rho_t U^2, \tag{1}$$

where ρ_j is the jet density; ρ_t, the target density; V_j, the velocity of the jet at the stagnation point; $U = dP/dt$, the penetration velocity; and λ, a parameter introduced to account for differences in the transfer of momentum to the target from a continuous jet and from a jet of discrete particles. The accuracy of the penetration equation has been extensively investigated by

* The existence of independent work by Hill, Mott, and Pack was cited in reference [1].

Eichelberger[4], who further generalized (1) by including an additive strength term. Attempts to use (1), or some modification of it, to correlate experimental values of U and V_j have been extremely successful.

By introducing plausible assumptions concerning the behavior of the jet in flight, Pugh[1] was able to integrate the penetration velocity and account for qualitative features of the penetration-standoff curve. This method requires defining an appropriate penetration velocity U for the penetration produced by the jet after it has broken into a series of discrete particles. Because the correct definition of U is not immediately obvious, this report describes a different approach to the problem in which the penetration process is represented by different mathematical models depending on whether the jet is continuous or broken-up.

AN APPROXIMATE SOLUTION FOR THE PENETRATION VARIABLES

To calculate the total depth of penetration in a massive target, it is necessary to add the penetrations produced by the various jet elements. For a continuous jet, correct values are obtained by integrating the penetration equation. After the jet fractures into a series of discrete particles, the physical process is better regarded as an addition of discrete elements of penetration corresponding to discrete particles of jet, and a more appropriate mathematical description is a summation of finite increments. Accordingly, the total penetration P_T is given by

$$P_T = \int_{t_o}^{t_1} U dt + \sum_{i=1}^{N} \Delta P_i, \tag{2}$$

where t_o is the time at which the jet reaches the target; t_1, the time at which the jet breaks up; and ΔP_i, the penetration produced by the i^{th} particle.

For a standoff of two or more cone diameters, it is well known that the velocity of the forward tip of the jet emerging from a target of thickness P, $(0 \leq P \leq P_T)$, can be calculated (to within a few per cent) from penetration time measurements by assuming that all jet elements are formed simultaneously at a virtual origin. Therefore an approximate solution for the penetration variables can be obtained by assuming that, at t = 0, the jet is formed at a

virtual origin located a distance z_o in front of the target surface. As shown in Figure 2(a), the penetration depth and impinging jet velocity are related by

$$P(t) = tV_j(t) - z_o \ , \tag{3}$$

where $V_j(t)$ is the velocity of the jet at the stagnation point. It is further assumed that the effects of strength are negligible; hence, for a continuous jet ($\lambda = 1$), Equation (1) reduces to

$$U = V_j/(1 + \gamma), \tag{4}$$

where $\gamma = (\rho_t/\rho_j)^{1/2}$. When (3) is differentiated with respect to time, one obtains

$$U = dP/dt = V_j + tdV_j/dt. \tag{5}$$

After the penetration velocity is eliminated between (4) and (5), the resulting equation in V_j and t can be integrated to give

$$V_j = V_j^o(t_o/t)^{\gamma/(1 + \gamma)}, \tag{6}$$

where V_j^o is the velocity of the unimpeded foremost tip of the jet, and t_o is the time at which it first encounters the target, relative to t = 0 at the virtual origin. Equation (6), together with (3), can be used to compute penetration-times and emerging-jet-velocities once the constants V_j^o and t_o have been determined experimentally, where Equation (3) is now given by,

$$P(t) = V_j^o \ t \ (t_o/t)^{\ \gamma/(1 + \gamma)} \tag{7}$$

After a time t_1, the penetration process changes from one involving a continuous jet to one involving a jet of discrete particles. An experimental determination of t_1 can be obtained from the point at which the penetration-time curve departs from that predicted for a continuous jet. If it is assumed that break-up occurs simultaneously throughout the remaining jet, then, as shown in Figure 2(b), the following relation must hold:

$$\sum_{i=1}^{N} \Delta\ell_i = z_o + P(t_1) - t_1 V_j^{min} \ , \tag{8}$$

where V_j^{min} is the minimum jet velocity capable of penetrating the target material and $\sum_{i=1}^{N} \Delta\ell_i$ is the length of the effective jet that has not yet penetrated the target. To calculate the total penetration, it is assumed that each particle of a broken-up jet behaves like a short steady-state jet[1]; i.e., $\Delta P_i = \Delta\ell_i/\gamma$. According to (2) and (8) and the definition of ΔP_i, the total penetration will be given by

$$P_T = P(t_1) + \left[z_o + P(t_1) - t_1 V_j^{min}\right] /\gamma, \tag{9}$$

which is independent of the total number of jet particles involved.

Flash radiographs indicate that the broken jet can be regarded as a series of discrete particles of approximately equal length[5]. If it is assumed that $\Delta\ell_1 = \Delta\ell_2 = \ldots \Delta\ell_N = \Delta\ell$, the number of effective jet particles will be given by

$$N = \left[z_o + P(t_1) - t_1 V_j^{min}\right] /\Delta\ell. \tag{10}$$

Flash radiographs also show that the increment in velocity between adjacent particles is approximately a constant[5]. Hence, the increment in velocity between adjacent particles is given by

$$\Delta V_j = -\left[V_j(t_1) - V_j^{min}\right] /(N - 1) \tag{11}$$

and the velocity of the i^{th} particle is given by

$$(V_j)_i = V_j(t_1) + (i - 1)\Delta V_j. \tag{12}$$

An inspection of Figure 2(c) shows that the time between the arrival of the i^{th} and the $(i + 1)^{st}$ particle is given by

$$\Delta t_{i,i+1} = \left[\Delta P + \Delta\ell - \Delta V_j(t_i - t_1)\right] /(V_j)_{i+1}. \tag{13}$$

Of course, the arrival time of the i^{th} particle is given by

$$t_i = t_1 + \sum_{j=2}^{i} \Delta t_{j-1,j} , \tag{14}$$

where $i \geq 2$.

The penetration depth at the instant the i^{th} particle reaches the bottom of the hole is given by

$$P(t_i) = P(t_1) + (i - 1)\Delta P. \qquad (15)$$

The penetration by the discontinuous portion of the jet will be completely specified once the constants $\Delta \ell$ and V_j^{min} have been determined. The number of particles is determined from (10), and their respective velocities are determined from (11) and (12). The time at which each particle starts to penetrate the target can be determined from (13) and (14), and the corresponding penetration depths can be obtained from (15).

The accuracy of the theory for correlating the penetration variables is best illustrated by analyzing data obtained with the 105 mm unconfined test charge used at the Ballistic Research Laboratories. The time at which fracture of the jet occurs was determined by integrating the hydrodynamic penetration equation for a continuous jet and comparing it with the experimental penetration-time data*. The point at which the two diverge; i.e., the time when the continuous jet approximation no longer adequately describes the penetration process, is taken to be the time at which simultaneous fracture occurs in the remainder of the effective jet. As shown in Figure 3, the divergence of the experimental data from the continuous jet approximation occurs at 120 μ-sec. The value of $\Delta \ell$ was estimated from flash radiographs to be 1.68 cm. The values of V_j^{min} and V_j^{o} were measured directly and found to be 0.210 cm/μ-sec and 0.701 cm/μ-sec, respectively.

The penetration theory for discrete particles can be used to extend the penetration-time curve beyond 120 μ-sec. As shown in Figure 3, results of the calculation are in good agreement with experimental observations. In addition to the penetration-time data, the velocity of jets emerging from targets of various thicknesses gives an independent measure of the jet velocity as a function of the penetration depth. As illustrated in Figure 4, the curve computed from the penetration theory adequately represents the emerging jet velocity measurements.

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* See reference 4, for a general description of the experimental techniques used in obtaining penetration-time data.

THE USE OF CONTINUOUS VARIABLES TO REPRESENT THE PENETRATION PRODUCED BY A PARTICLE JET

It has already been noted that the total penetration produced by the broken-up portion of the jet depends on the total length of all the particles involved, but not on the length of an individual particle. It is therefore permissible to represent the penetration variables by continuous functions obtained from appropriate mathematical limits taken as the particle length $\Delta\ell$ approaches zero and the number of particles N approaches infinity. Of course, the limits must be taken in such a way that

$$L = \lim_{\substack{\Delta\ell \to 0 \\ N \to \infty}} (N\Delta\ell) = z_o + P(t_1) - t_1 V_j^{min} \tag{16}$$

is a constant.

An expression for the velocity of the impinging jet is obtained after dividing (11) by $\Delta\ell$,

$$\Delta V_j/\Delta\ell = - \left[V_j(t_1) - V_j^{min}\right] /(N\Delta\ell - \Delta\ell), \tag{17}$$

taking the limit as $\Delta\ell$ tends to zero,

$$\lim_{\Delta\ell \to 0} (\Delta V_j/\Delta\ell) = dV_j/d\ell = - \left[V_j(t_1) - V_j^{min}\right] /L \tag{18}$$

and integrating,

$$\int_{V_j(t_1)}^{V_j(s)} dV_j = - (1/L) \left[V_j(t_1) - V_j^{min}\right] \int_0^s d\ell. \tag{19}$$

The final expression for the jet velocity is

$$V_j(s) = V_j(t_1) - \left[V_j(t_1) - V_j^{min}\right] s/L, \tag{20}$$

where the parameter $s = \int_0^s d\ell$ is the location of an infinitesimal element of jet relative to the stagnation point at time t_1.

An expression for the penetration is readily obtained by taking the limit of ΔP_i as $\Delta \ell$ tends to zero,

$$\lim_{\Delta \ell \to 0} (\Delta P/\Delta \ell) = dP/d\ell = 1/\gamma \tag{21}$$

and integrating,

$$\int_{P(t_1)}^{P(s)} dP = (1/\gamma)\int_0^s d\ell. \tag{22}$$

The resulting expression for the penetration is

$$P(s) = P(t_1) + s/\gamma. \tag{23}$$

In much the same way, an expression for the time is obtained after dividing (13) by $\Delta \ell$.

$$(V_j)_{i+1}(\Delta t_{i,i+1}/\Delta \ell) = (\Delta P/\Delta \ell) + 1 - (\Delta V_j/\Delta \ell)(t_i - t_1) , \tag{24}$$

taking the limit as $\Delta \ell$ tends to zero,

$$V_j dt/d\ell = (1/\gamma) + 1 - (t_i - t_1)dV_j/d\ell , \tag{25}$$

and integrating

$$\int_{V_j(t_1)t_1}^{V_j(s)t(s)} d(V_j t) = \frac{(1 + \gamma)}{\gamma}\int_0^s d\ell + t_1 \int_{V_j(t_1)}^{V_j(s)} dV_j. \tag{26}$$

The final expression for the time is

$$t(s) = t_1 + s(\gamma + 1)/\gamma V_j(s). \tag{27}$$

Equations (20), (23), and (27) are parametric equations from which the penetration variables can be calculated. The parameter is permitted to vary continuously over the range $0 \leq s \leq L$, and the penetration variables calculated from the equations are continuous functions. A graph illustrating the functional relation between the penetration and the time is presented in Figure 5. For comparison purposes, results computed from the discrete particle theory have been superimposed. The time and penetration depth corresponding to each

particle impact on the target were computed from (14) and (15) respectively. To emphasize the discrete nature of the process, the time required for each particle to complete its penetration was computed from

$$\Delta t_i = \Delta P_i/U_i = (1 + \gamma)\ P_i/(V_j)_i\ . \qquad (28)$$

According to the idealized model of the particle jet, there will be intervals of time $(\Delta t_{i,i+1} - \Delta t_i)$ during which no penetration takes place*. The length of this interval increases with the depth of penetration because adjacent particles have more time to spread due to differences in their velocities. As shown in Figure 5, the continuous parameter representation of the penetration-time curve follows the discrete particle theory very closely. Because the equations are simpler, future calculations of penetration variables will use the continuous parameter representation.

For a continuous jet Bernoulli's equation was used to establish a relation between the two velocities V_j and U[1]. A similar relation can be obtained for a jet of discrete particles by differentiating (20), (23) and (27) with respect to t, obtaining

$$dV_j/dt = -\left[V_j(t_1) - V_j^{min}\right](ds/dt)/L\ , \qquad (29)$$

$$U = dP/dt = (1/\gamma)\ ds/dt\ , \qquad (30)$$

$$(t - t_1)dV_j/dt + V_j = \left[(\gamma + 1)/\gamma\right] ds/dt \qquad (31)$$

respectively. When (30) is used to eliminate ds/dt from (29) and (31), one obtains,

$$dV_j/dt = -\left[V_j(t_1) - V_j^{min}\right]\gamma U/L\ , \qquad (32)$$

$$(t - t_1)dV_j/dt + V_j = (\gamma + 1)U. \qquad (33)$$

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* The discontinuous nature of the penetration variables was neglected in Figure 3 and Figure 4, a smooth curve being drawn through points representing the variable at the instant each particle impacts with the target.

Elimination of dV_j/dt between (32) and (33) gives

$$- \left[V_j(t_1) - V_j^{min}\right] \gamma U(t - t_1)/L + V_j = (\gamma + 1)U. \tag{34}$$

To eliminate $(t - t_1)$ from the preceding equations, s is eliminated between (20) and (27) to give

$$(t - t_1) = (\gamma + 1)L \left[V_j(t_1) - V_j\right] / \gamma V_j \left[V_j(t_1) - V_j^{min}\right] . \tag{35}$$

Finally, substitution of (35) into (34) gives

$$U = V_j^2/(\gamma + 1)V_j(t_1). \tag{36}$$

For a jet of discrete particles, the penetration velocity is proportional to the square of the jet velocity, and for this reason, the properties of a discontinuous jet cannot be described by replacing ρ_j in Equation (1) with an effective density. It is interesting to note that the problem of defining an appropriate penetration velocity for a discontinuous jet has been solved, and the penetration can be obtained by integrating the penetration velocity; i.e.,

$$P(t) = \int_{t_o}^{t} U dt = \int_{t_o}^{t_1} U_1 dt + \int_{t_1}^{t} U_2 dt, \ (t \geq t_1). \tag{37}$$

Where U_1 is given by (4) and U_2 by (36).

SCALING RELATIONS

It has been shown experimentally that the penetration-time curves for a homologous series of scaled charges reduce to a single curve when both the penetration and time are divided by the charge diameter[5]. The accuracy with which a single curve represents the penetration-time data is illustrated in Figure 6. For the data presented in Figure 6, the location of the virtual origin with respect to the top of the target can be expressed as $z_o = \zeta_o D$, where D is the charge diameter and ζ_o, a constant.

Equations (3) and (6) can be used to show that the penetration, measured in charge diameters, is given by

$$P/D = \pi(\tau) = \tau V_j - \zeta_o , \tag{38}$$

where

$$V_j = V_j^{\circ} \ (\tau_o/\tau)^{(\gamma/1 + \gamma)} \tag{39}$$

and

$$\tau = t/D. \tag{40}$$

After the jet breaks up the penetration can be expressed as a function of the dimensionless parameter $\sigma = s/D$. The penetration, measured in charge diameters, is given by

$$P/D = \pi(\sigma) = \pi(\tau_1) + \sigma/\gamma \tag{41}$$

and the corresponding times are given by

$$t/D = \tau(\sigma) = \tau_1 + \sigma(\gamma + 1)/\gamma V_j \ , \tag{42}$$

where V_j is now given by

$$V_j = V_j(\tau_1) - \left[V_j(\tau_1) - V_j^{min}\right] \ \sigma/\left[\zeta_o + \pi(\tau_1) - \tau_1 V_j^{min}\right] \ . \tag{43}$$

The preceding equations, (38) to (43), describe the penetration of a shaped-charge jet in terms of size independent variables, $\tau = t/D$ and $\pi = P/D$, and are obtained from Equations (16), (20), (23) and (27). For a homologous series of scaled charges there exists a unique relation between π and τ, which is independent of the charge size. The penetration velocity U and the jet velocity V_j can also be represented as functions of either π or τ, the functions being independent of charge size.

GENERALIZATION OF THE THEORY FOR SHORT STANDOFF

The present theory will be inaccurate for short standoffs where it is no longer permissible to neglect small differences between the times and positions at which different jet elements are formed. To remove the inaccuracies at short standoffs, a complete description of the liner-collapse and jet-formation process is required. At the present time the necessary data are available for only one charge, the 105 mm unconfined test charge used at the Ballistic Research Laboratories[6]. For this charge, the collapse velocity in cm/μ-sec imparted to an element of the liner is given by

$$V_o = -0.197898 + 0.288477x - 0.0775152x^2 + 0.00942615x^3 - 0.000445364x^4, \quad \text{for } 4.0 \text{ cm} \leq x \leq 8.0 \text{ cm}, \tag{44}$$

where x is the axial position of the liner element measured relative to the apex of the conical liner.

Equations describing a point initiated charge of length L containing a conical cavity of height H are readily derived using the quantities illustrated in Figure 7. The zero reference for time, t = 0, is taken to be the instant the explosive is initiated. The time required for the detonation wave to reach a liner element located at the position x is given by

$$T(x) = \left[(L - H + x)^2 + (x \tan \alpha)^2\right]^{1/2}/U_D , \tag{45}$$

where U_D is the detonation velocity of the explosive. The time at which the liner element reaches the axis is obtained from

$$t^+ (x) = T(x) + x \tan \alpha/V_o \cos (\alpha + \delta) \tag{46}$$

and the point at which the liner element reaches the axis is given by

$$z^+(x) = x + (t^+ - T)V_o \sin (\alpha + \delta). \tag{47}$$

Values for the angle δ can be computed from

$$\delta = \sin^{-1}(V_o \cos \epsilon /2U_D), \tag{48}$$

where $$\tan(90^o - \alpha + \epsilon) = (L - H + x)/(x \tan \alpha). \tag{49}$$

The depth of penetration at a time t will be given by

$$P(t) = (t - t^+)V_j(t) - (S + H - z^+) , \tag{50}$$

where S is the standoff between the base of the charge and the top of the target. The jet velocity, $V_j(t)$, appearing in the penetration equation is the value for the jet element at the stagnation point between the jet and target. The jet velocity may also be regarded as a function of x, the original position of the element within the liner. The function $V_j(x)$ can be computed from the non-steady theory of jet formation[2]. The penetration velocity obtained by differentiating (50) is

$$U = dP/dt = V_j + \left[t - Q(V_j)\right] (dV_j/dt) , \tag{51}$$

where

$$Q(V_j) = (t^+ + V_j\, dt^+/dV_j - dz^+/dV_j). \tag{52}$$

The differential equation obtained by eliminating U between (51) and (4) is

$$dV_j/dt = -\gamma V_j/(1 + \gamma) \left[t - Q(V_j)\right] . \tag{53}$$

The virtual origin approximation is obtained by letting $Q(V_j)$ be zero. Equation (53) is a first order linear differential equation of the form $dV_j/dt = f(V_j, t)$ and can be solved by well known numerical procedures.

The numerical solution to the differential equation (53) requires a precise knowledge of the cone collapse parameters. The data presently available on the 105mm unconfined test charge, while adequately describing the jet formation, do not provide sufficiently precise initial conditions necessary to solve the short standoff equation. In lieu of using the generalization, the virtual origin approximation was shown to be satisfactory for standoffs down to half of a cone diameter. Hence the generalization reduces to one of academic interest, and will be resolved when more precise data becomes available.

EFFECTS OF COMPRESSIBILITY

As pointed out in the introduction, the penetration of a target by a shaped charge jet is described by incompressible fluid flow. However, compressibilities of the target and jet are certainly not negligible at the pressures produced during the penetration, and it is important to estimate the error introduced by the incompressible approximation.

Consider the steady state penetration of a compressible target by a compressible jet in which the jet and target materials approach the stagnation point with subsonic velocities. For adiabatic compressible flow, Bernoulli's equation requires that

$$\int_0^{p_s} \frac{\rho_t^o}{\rho_t}\, dp - (1/2)\rho_t^o U^2 = 0, \tag{54}$$

$$\int_0^{p_s} \frac{\rho_j^o}{\rho_j}\, dp - (1/2)\, \rho_j^o (V_j - U)^2 = 0 \tag{55}$$

along the axial streamline, where p_s is the stagnation pressure; and ρ^o, the respective densities at normal pressure. The effect of compressibility on the penetration velocity is readily determined by setting the left-hand side of (54) equal to the left-hand side of (55) and rearranging terms. Thus one obtains

$$(1/2)\rho_j^o\ (V_j - U)^2 - (1/2)\rho_t^o U^2 = \int_0^{p_s} \left[(\rho_j^o/\rho_j) - (\rho_t^o/\rho_t)\right] dp. \qquad (56)$$

If the material is incompressible the integral in (56) is zero and the resulting expression is equivalent to (1) with λ set equal to one. It is apparent that the integral is zero if the jet and target are the same material. On the other hand, the largest error will result from treating as incompressible two materials that have greatly different compressibilities.

It is possible to compute the error for any combination of materials for which suitable equation-of-state data are available. Usually, the high pressure dynamic measurements provide an empirical relation between the pressure and density along the shock Hugoniot. When the Grüneisen ratio is also known, the pressure and density can be calculated for any adiabat that intersects the Hugoniot[+]. The penetration of a titanium target by a copper jet was chosen as an illustration because titanium is considerably more compressible than copper. Calculations were made using the equation of state data reported by McQueen and Marsh[7]. Assuming a stagnation pressure of 500 kilobars, a penetration velocity of 0.412 cm/μ-sec is computed from (54), and a jet velocity of 0.726 cm/μ-sec is obtained from (55). For a penetration velocity of 0.412 cm/μ-sec, the incompressible approximation predicts a jet velocity of 0.720 cm/μ-sec, which is within 1% of the more exact calculations. The relative magnitudes of the quantities appearing in (56) are illustrated graphically in Figure 8. The difference between $(1/2)\rho_j^o(V_j - U)^2$ and $(1/2)\rho_t^o U^2$, which is represented by the

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+ For a general discussion concerning the behavior of solids at high dynamic pressures, the reader is referred to the section entitled "Compression of Solids by Strong Shock Waves" contributed by M. H. Rice, R. G. McQueen, and J. M. Walsh to Solid State Physics, Vol. 6, Edited by Fredrick Seitz and David Turnbull, Academic Press, Inc. Publishers (1958).

shaded area, is a small fraction of $(1/2)\ \rho_t^o U^2$, which is represented by the cross hatched area. It is noted, however, that the pressure computed from the incompressible approximation, $(1/2)\ \rho_t^o U^2$, is 422 kilobars, or 16% less than the actual pressure. Thus, as already demonstrated by direct experimental observations, the relationship between the jet and penetration velocities obtained from the incompressible approximation is sufficiently accurate for all practical problems involving the penetration of shaped-charge jets.

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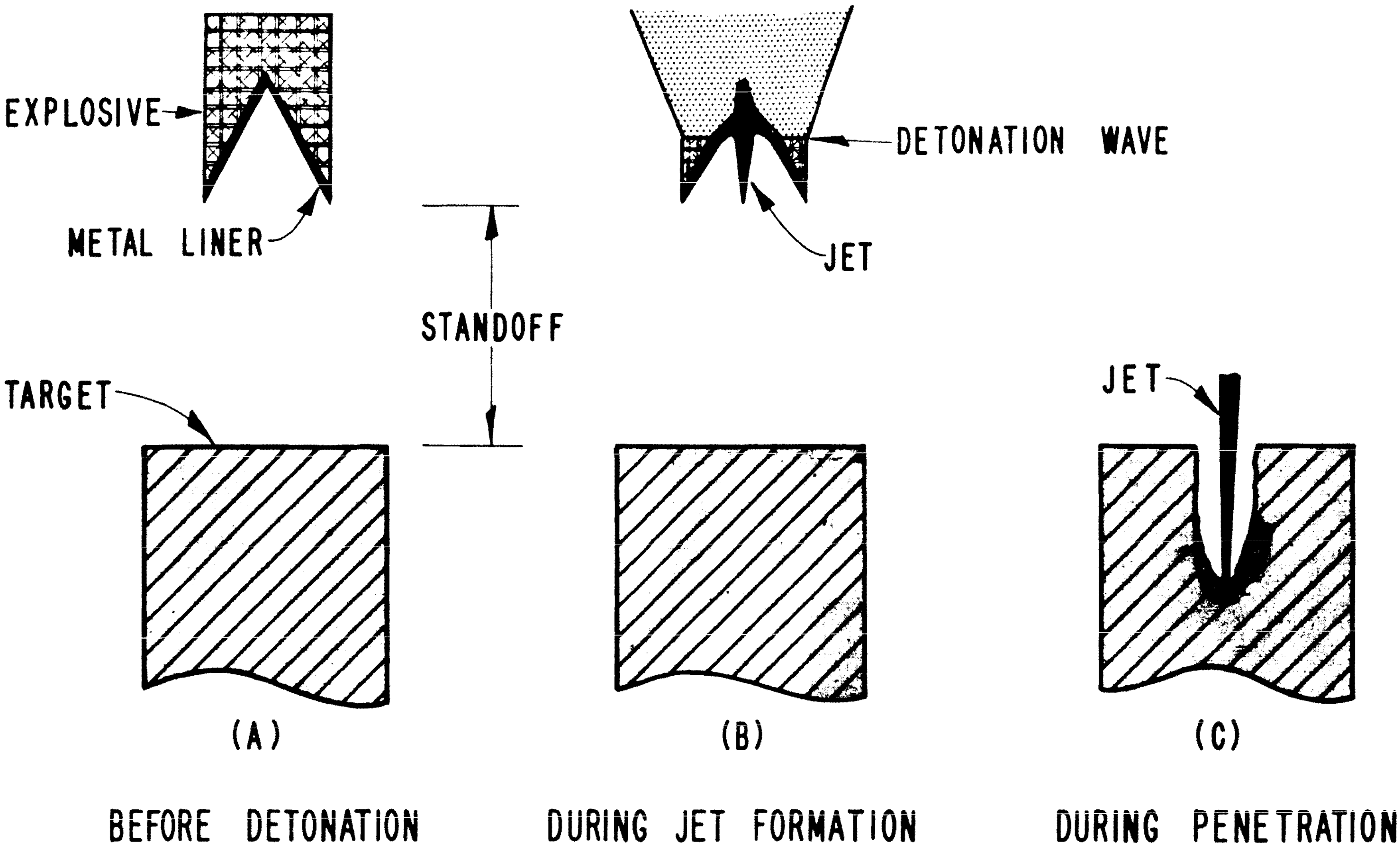

Fig. 1 Line drawings illustrating the formation of a shaped-charge jet and the subsequent penetration of a target by the jet. Note the hydrodynamic nature of the penetration process.

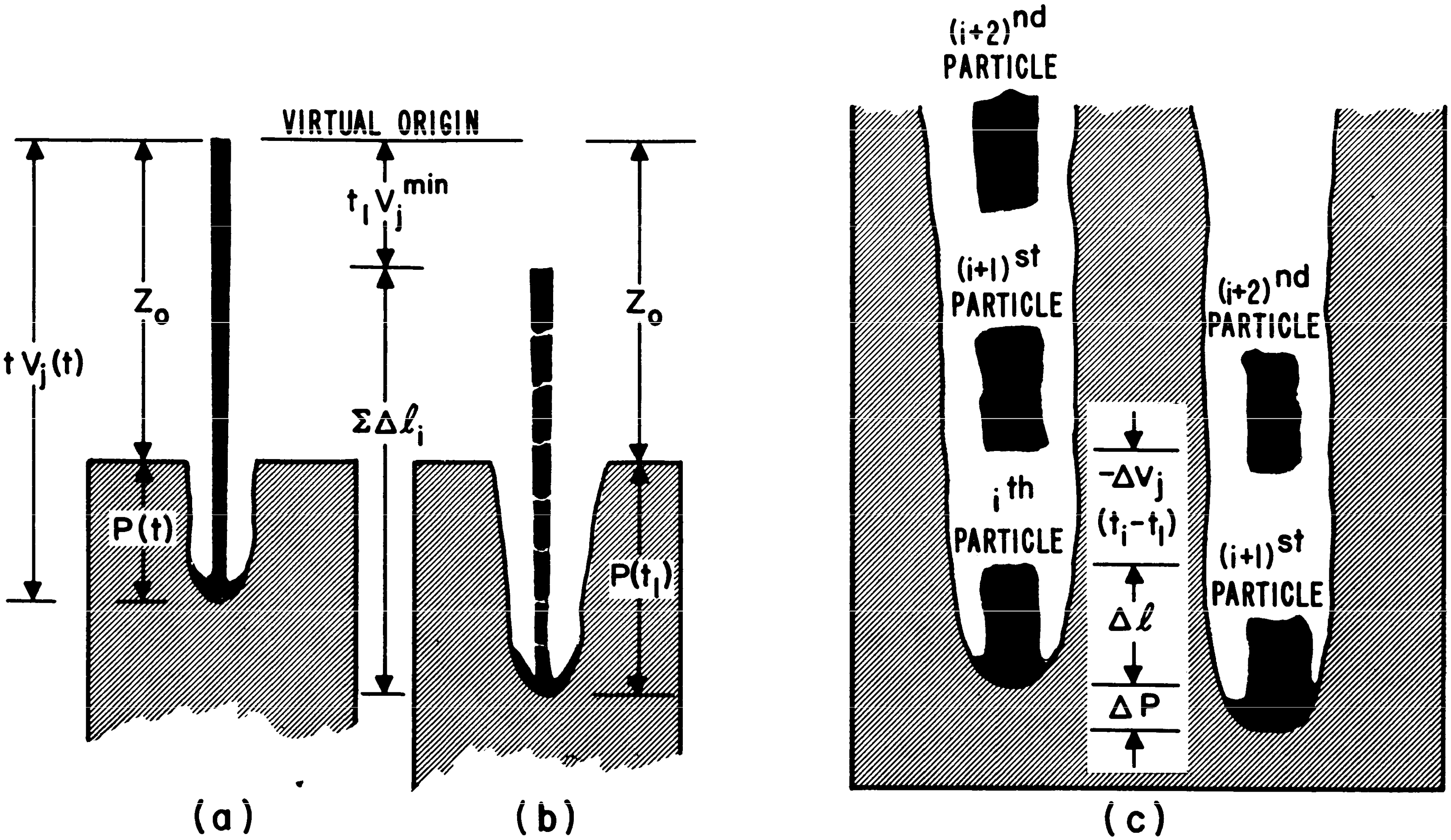

Fig. 2 Diagram illustrating: (a) the virtual origin approximation (b) the relationship between the penetration variables at the instant of jet break-up, and (c) the time interval between the impact of the ith and the (i + 1)st particles.

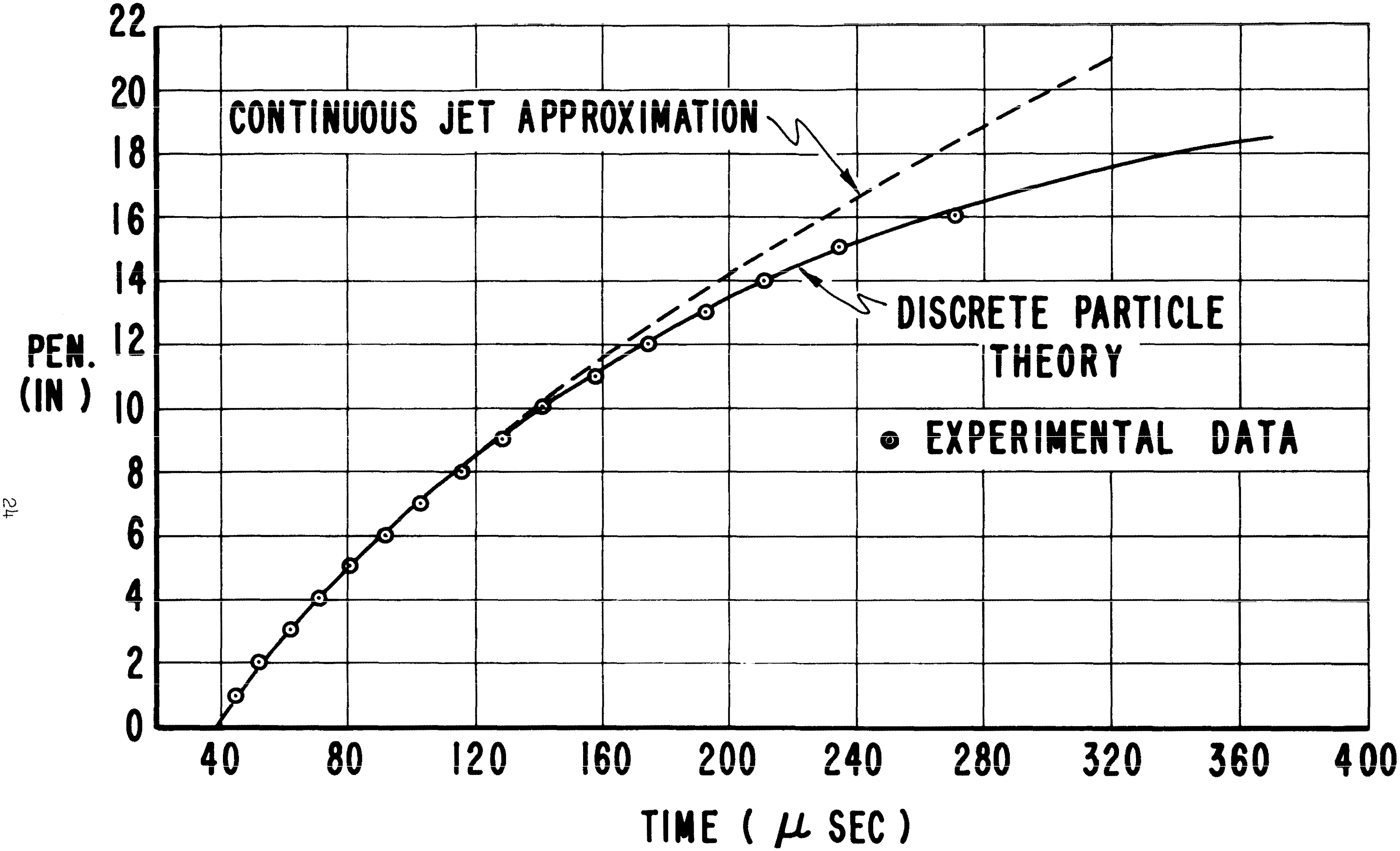

Fig. 3 Penetration-time curve for the 105mm unconfined test charge fired at an 8 1/4 in. standoff. Note that the continuous jet approximation is valid for the first 120μ-sec. Beyond this point, the penetration time must be calculated using a method applicable to a jet of discrete particles.

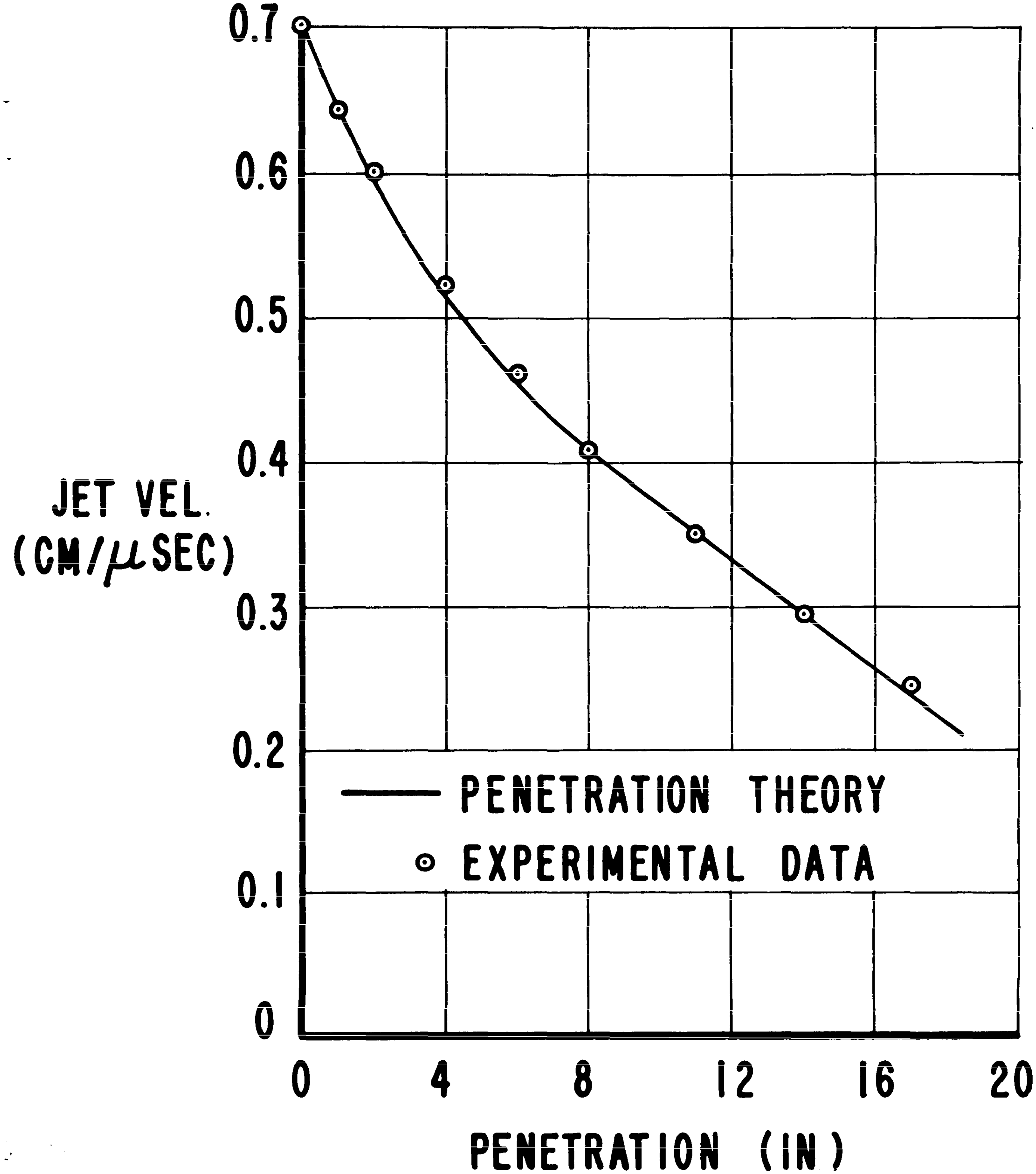

Fig. 4 Emerging jet velocity versus the penetration depth for the 105mm unconfined test charge fired at an 8 1/4 in. standoff. The theoretical curve was computed from the penetration theory developed in this report.

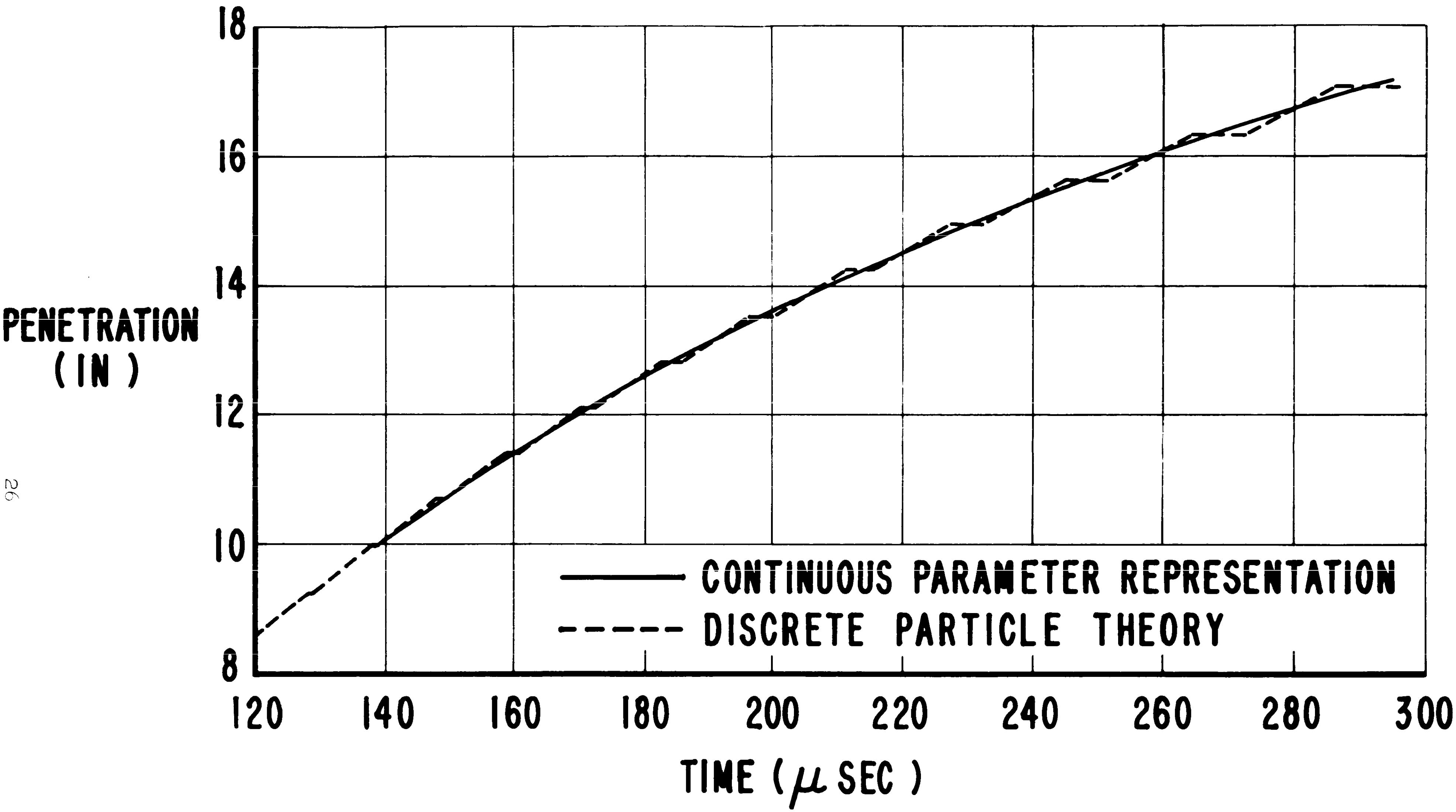

Fig. 5 Curves representing the penetration produced by the broken-up portion of jets obtained from 105mm unconfined test charges fired at an 8 1/4 in. standoff.

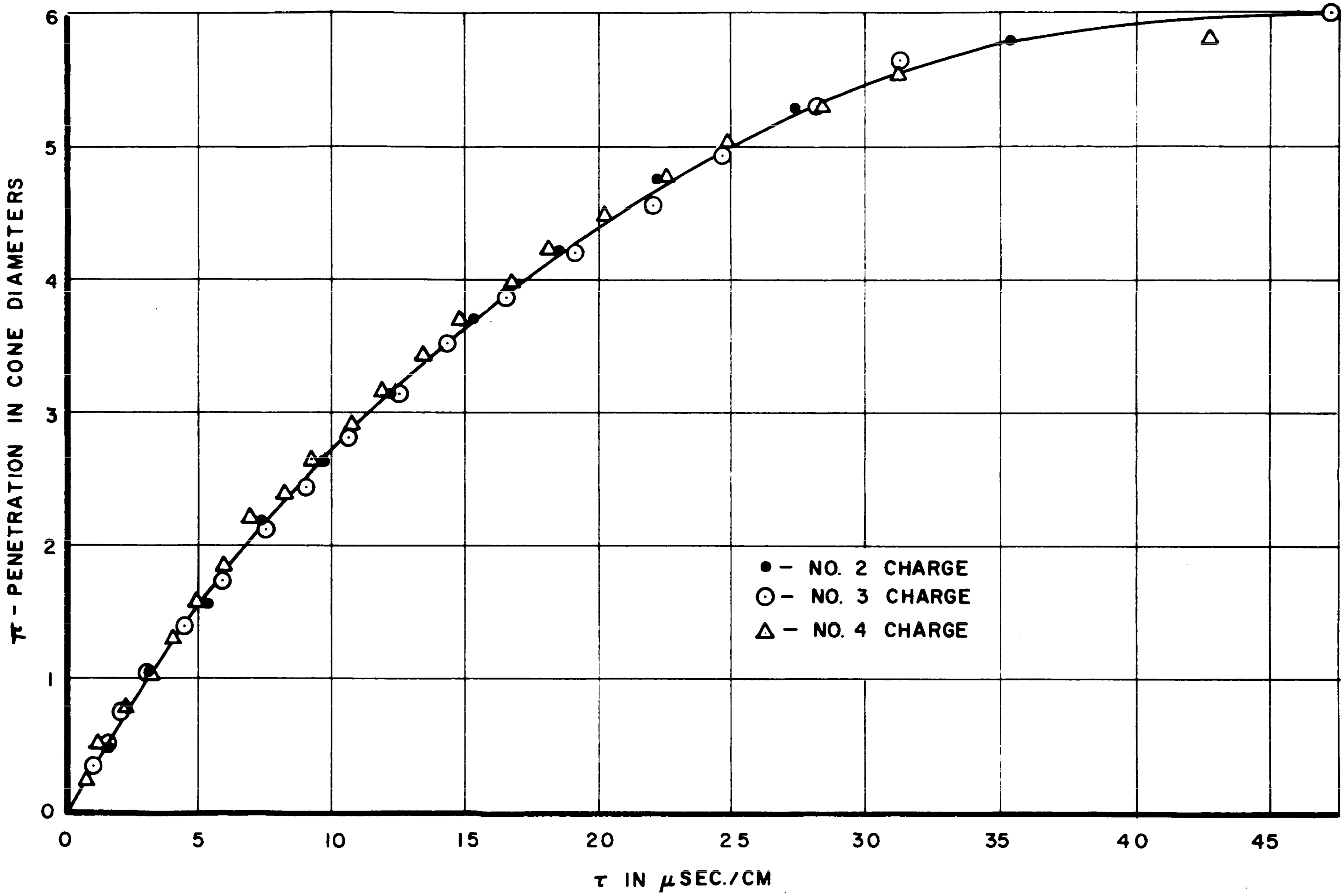

Fig. 6 Graph showing that the penetration-time data for a homologous series of scaled charges can be represented by a single curve when both the penetration and time are divided by a characteristic linear dimension, such as the charge diameter.[5]

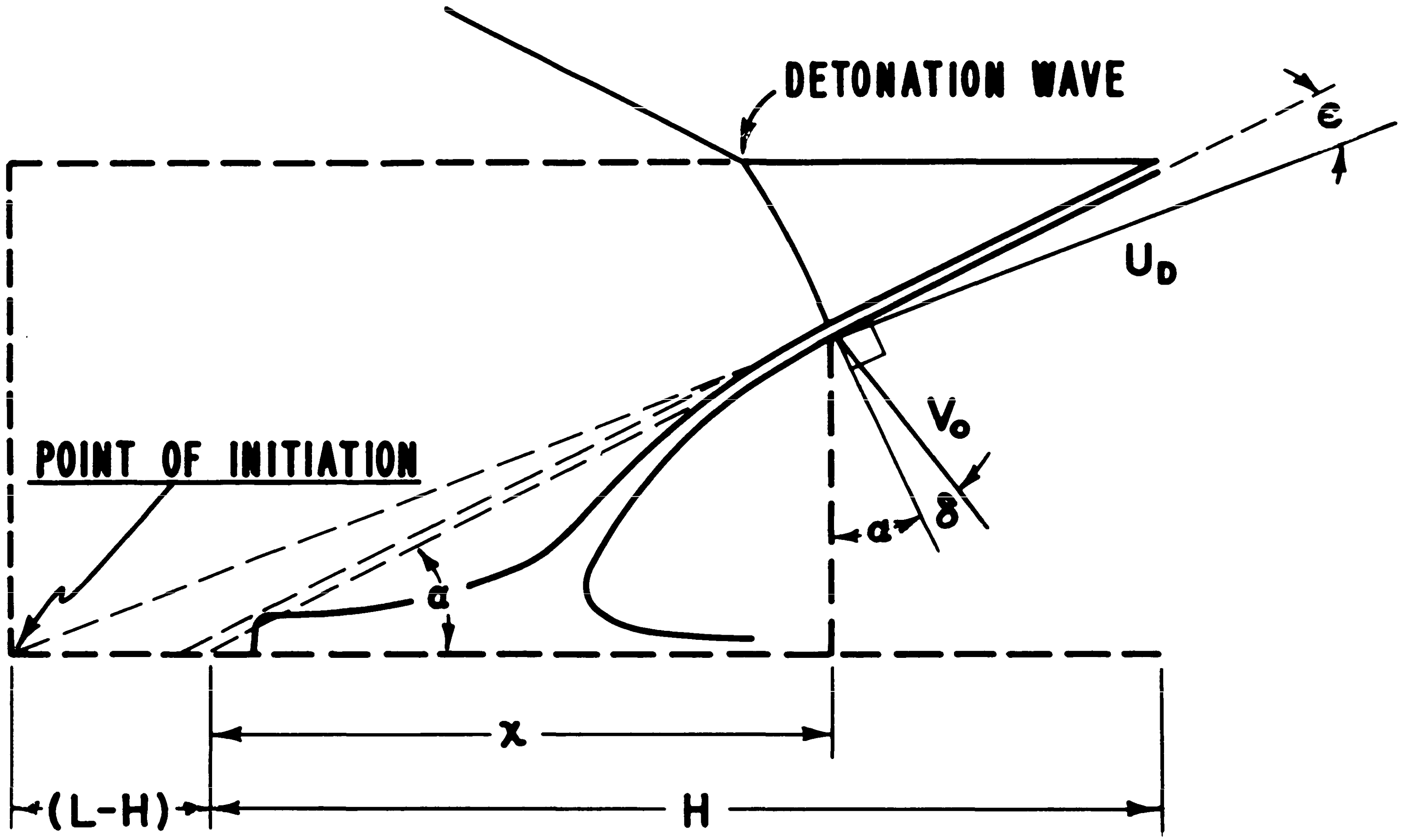

Fig. 7 Line drawing illustrating the quantities employed in the hydrodynamic description of the collapse process for a lined conical cavity.

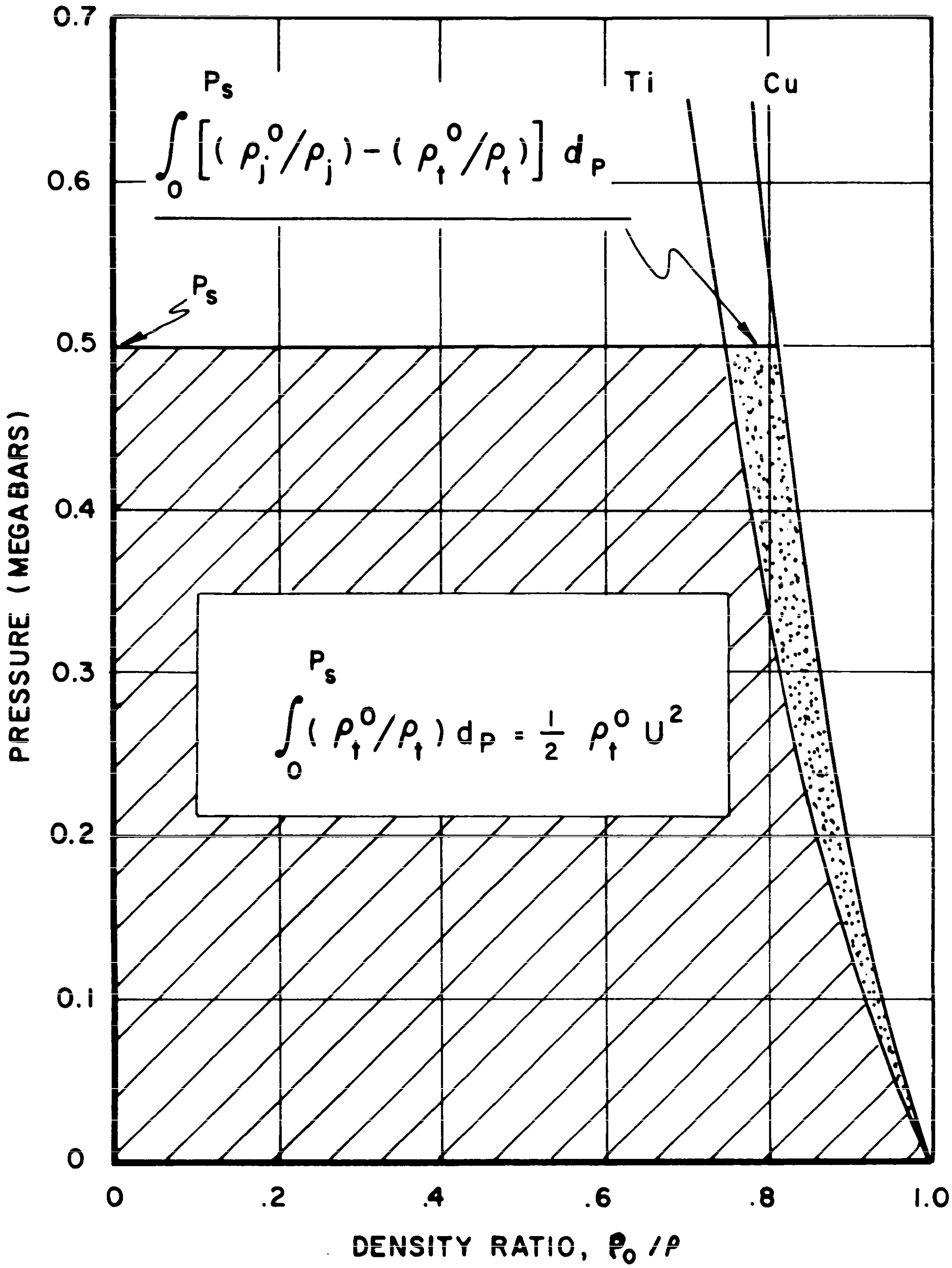

Fig. 8 Graphical illustration of the relative magnitudes of the quantities resulting from the compressible and incompressible treatment of Ti penetrating Cu.

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